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On S- ρ -Connected Space in a Topological Space

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ABSTRACT
The authors introduced S-L-open sets, S-M-closed sets, S-R-open sets and S-S-closed sets and established their relationships with some generalized setsin a topological spaces. Connected spaces constitute the most important classes of topological spaces. In this paper we introduce the concept "S- ρ -connected" in a topological space.
Keywords: *S-L-open sets, S-M-closed sets, S-R-open sets, S-S-closed sets,S-L-connected space, S-M-connected space, S-R-connected space, S-S-connected space, S-L-irresolute,S-M- irresolute,S-R-resolute,S-S-resolute.*

1. INTRODUCTION

Intopology and related branches of mathematics a connected space is a topological spaces that cannot be represented as the union of two disjoint non empty open subsets. Connectedness is one of the principal topological properties that are used to distinguish topological spaces. In this paper we introduce S- ρ -connected spaces. A topological space X is said to be S- ρ -connected if X cannot be written as the disjoint union of two non-empty S- ρ -open (S- ρ -closed) sets in X.

2. PRELIMINARIES

Throughout this paper $f^{-1}(f(A))$ is denoted by A^* and $f(f^{-1}(B))$ is denoted by B^*.

Definition 2.1

Let A be a subset of a topological space (X, τ).Then A is called semi-open if A $\subseteq$ cl(int(A)) and semi-closed if int (cl(A)) $\subseteq$ A; [1].

Definition 2.2

Let f: (X, τ) $\rightarrow$ (Y, σ) be a function. Then f is semi-continuous if f^{-1}(B) is open in X for every semi-open set B in Y. [1]

Definition: 2.3

Let f: (X, τ) $\rightarrow$ (Y, σ) be a function. Then f is semi-open (resp. semi-closed) if f(A) is semi-open(resp. semi-closed) in Y for every semi-open(resp. semi-closed) set A in X.[1]

Definition: 2.4

Let f: (X, τ) $\rightarrow$ Y be a function. Then f is

1. S-L-Continuous if A^* is open in X for every semi-open set A in X.
2. S-M-Continuous if A^* is closed in X for every semi-closed set A in X.[2]

Definition: 2.5

Let f: X$\rightarrow$(Y,σ) be a function. Then f is

1. S-R-Continuous if B^* is open in Y for every semi-open set B in Y.
2. S-S-Continuous if B^* is closed in Y for every semi-closed set B in Y.[2]

Definition: 2.6

Let f: (X,τ) $\rightarrow$(Y,σ) be a function, then f is said to be

1. S-irresolute if $f^{-1}(V)$ is semi-open in X, whenever V is semi-open in Y.
2. S-resolute if $f(V)$ is semi-open in Y, whenever V is semi-open in X.[7]

Definition: 2.7

Let (X,τ) is said to be

1) Finitely S-additive if finite union of semi-closed set is semi-closed.
2) Countably S-additive if countable union of semi-closed set is semi-closed.
3) S-additive if arbitrary union of semi-closed set is semi-closed.[9]

Definition: 2.8 Let (X,τ)be a topological space and x$\in$X .Every semi-open set containing x is said to be a S-neighbourhood of x.[6]

Definition: 2.9 Let A be a subset of X. A point x$\in$X is said to be semi-limit point of A if every semi-neighbourhood of x contains a point of A other than x. [6]

Definition: 2.10

X=A$\cup$B is said to be a semi-separation of X if A and B are non-empty disjoint and semi-open sets .If there is no semi-separation of X, then X is said to be S-connected. Otherwise it is said to be S-disconnected. [7]

3. S- ρ -OPEN (CLOSED) SETS

Definition: 3.1

Let f: (X,τ)$\rightarrow$Y be a function and A be a subset of a topological space (X, τ).Then A is called

1. S-L-open if $A^* \subseteq cl(\text{int}(A^*))$
2. S-M-closed if $A^* \supseteq \text{int}(\text{cl}(A^*))$

Definition: 3.2

Let f: X$\rightarrow$(Y,σ) be a function and B be a subset of a topological space (Y,σ). Then B is called

1. S-R-open if $\text{B}^* \subseteq cl(\text{int}(\text{B}^*))$
2. S-S-closed if $\text{B}^* \supseteq \text{int}(\text{cl}(\text{B}^*))$

Example: 3.3

Let X = {a, b, c } and Y = {1, 2, 3}. Letτ = { Φ,X ,{a}, {b}, {a, b} }. Let f: (X,τ)$\rightarrow$Y defined by f(a)=2, f(b)=1, f(c)=3. Then f is S-L-openand S-M-Closed.

Example: 3.4

Let X = {a, b, c} and Y = {1, 2, 3}. Let σ ={ Φ,Y, {1},{2},{1,2} }.Let g : X$\rightarrow$(Y,σ) defined by g(a)=2, g(b)=2, g(c)=3. Then g is S-R-open and S-S-Closed.

Definition: 3.5

Let f: (X,τ) $\rightarrow$(Y,σ) be a function, then f is said to be

1. S-L-irresolute if $f^{-1}(f(\text{A}))$ is semi-L-open in X, whenever A is semi-L-open in X.
2. S-M-irresolute if $f^{-1}(f(\text{A}))$ is semi-M-closed in X, whenever A is semi-M-closed in X.
3. S-R-resolute if $f(f^{-1}(\text{B}))$ is semi-R-open in Y, whenever B is semi-R-open in Y.
4. S-S-resolute if $f(f^{-1}(\text{B}))$ is semi-S-closed in Y, whenever B is semi-S-closed in Y.

Definition: 3.6

Let (X,τ) is said to be

1. finitely S-M additive if finite union of S-M-closed set is S-M-closed.
2. Countably S-M-additive if countable union of S-M-closed set is S-M-closed.
3. S-M-additive if arbitrary union of S-M-closed set is S-M-closed.

Definition: 3.7 Let (X,τ) be a topological space and $x \in X$ .Every semi-L-open set containing x is said to be a S-L-neighbourhood of x.

Definition: 3.8 Let A be a subset of X. A point $x \in X$ is said to be semi-L-limit point of A if every semi-L-neighbourhood of x contains a point of A other than x.

4. S-ρ -CONNECTED SPACES

Definition: 4.1

X=A$\cup$B is said to be a S-L-separation of X if A and B are non empty disjoint and S-L-open sets .If there is no S-L-separation of X, then X is said to be S-L-connected. Otherwise it is said to be S-L-disconnected.

Definition: 4.2

X=A$\cup$B is said to be a S-M-separation of X if A and B are non empty disjoint and S-M-closed sets. If there is no S-M-separation of X, then X is said to be S-M-connected. Otherwise it is said to be S-M-disconnected.

Example: 4.3

Let X = {a, b, c, d} and Y = {1, 2, 3, 4}. Let τ = { Φ ,X ,{a}, {b}, {a, b}, {a, b, c} }. Let f: (X,τ)$\rightarrow$Y defined by f(a)=1, f(b)=1, f(c)=2, f(d)=2. Then X is S-L-connected space and S-M-connected space.

Definition: 4.4

X=A$\cup$B is said to be a S-R-separation of X if A and B are non empty disjoint and S-R-open sets .If there is no S-R-separation of X, then X is said to be S-R-connected. Otherwise it is said to be S-R-disconnected.

Definition: 4.5

X=A$\cup$B is said to be a S-S-separation of X if A and B are non empty disjoint and S-S-closed sets. If there is no S-S-separation of X, then X is said to be S-S-connected. Otherwise it is said to be S-S-disconnected.

Example: 4.6

Let X = {a, b, c, d} and Y = {1, 2, 3, 4}. Let σ ={ Φ,Y, {1},{2},{1,2},{1,2,3} }. Let g : X$\rightarrow$(Y,σ) defined by g(a)=1, g(b)=1, g(c)=2, g(d)=2. Then Y is S-R-connected space and S-S-connected space.

Note: 4.7

i. If X=A$\cup$B is a S-L-separation then A^C = B and B^C = A. Hence A and B are S-M-closed.
ii. If X=A$\cup$B is a S-M-separation then A^C = B and B^C = A. Hence A and B are S-L-open.
iii. If X=A$\cup$B is a S-R-separation then A^C = B and B^C = A. Hence A and B are S-S-closed.
iv. If X=A$\cup$B is a S-S-separation then A^C = B and B^C = A. Hence A and B are S-R-open.

Remark: 4.8

(X,τ) is S-L-connected (or) S-M-connected if and only if the only subsets which are both S-L-open and S-M-closed are X and ϕ.

Proof:

Let (X,τ) is S-L-connected space (or) S-M-connected space. Suppose that A is a proper subset which is both S-L-open and S-M-closed, then X=A$\cup$$A^c$ is a S-L-separation (S-M-separation) of X which is contradiction. Conversely, let ϕ be the only subsets which is both S-L-open and S-M-closed. Suppose X is not S-L-connected (not S-M-connected), then X=A$\cup$B where A and B are non empty disjoint S-L-open (S-M-closed) subsets which is contradiction.

Example: 4.9

Any indiscrete topological space (X,τ) with more than one point is not S-L-connected (not S-M-connected).X={a}$\cup${a}^{c}is a S-L-separation(S-M-separation), since every subset is S-L-open(S-M-closed).

Remark: 4.10

(Y,σ) is S-R-connected (or) S-S-connected if and only if the only subsets which are both S-R-open and S-S-closed are Y and ϕ.

Proof:

Let (Y,σ) is S-R-connected space (or) S-S-connected space. Suppose that A is a proper subset which is both S-R-open and S-S-closed, then Y= A$\cup$A ^{c} is a S-R-separation (S-S-separation) of Y which is contradiction. Conversely, let ϕ be the only subsets which is both S-R-open and S-S-closed. Suppose Y is not S-R-connected (not S-S-connected), then Y=A$\cup$B where A and B are non empty disjoint S-R-open (S-S-closed) subsets which is contradiction.

Example: 4.11

Any indiscrete topological space (Y,σ) with more than one point is not S-R-connected (not S-S-connected).Y={a}$\cup${a}^{c}is a S-R-separation(S-S-separation), since every subset isS-R-open(S-S-closed).

Theorem: 4.12

Every S-L-connected space is connected.

Proof:

Let X be an S-L-connected space. Suppose X is not connected. Then X=A$\cup$B is a separation then it is a S-L-separation. Since every open set is S-L-open set in X. This is contradiction to fact that (X,τ) is S-L-connected. Therefore hence (X,τ) is connected.

Remark: 4.13

The converse of the theorem (4.12) is not true as seen from example (4.14).

Example: 4.14

Any indiscrete topological space (X,τ) with more than one point is not S-L-connected. Since every subset is S-L-open. But it is connected, since the only open sets are X and ϕ.

Theorem: 4.15

Every S-M-connected space is connected.

Proof:

Let X be an S-M-connected space. Suppose X is not connected. Then X=A$\cup$B is a separation then it is a S-M-separation. Since every closed set is S-M-closed set in X. This is contradiction to fact that (X,τ) is S-M-connected. Therefore hence (X,τ) is connected.

Definition: 4.16

Let Y be a subset of X. Then Y=A$\cup$B is said to be a S-L-separation (S-M-separation) of Y if A and B are non empty disjoint S-L-open (S-M-closed) sets in X. If there is no S-L-separation (S-M-separation) of Y then Y is said to be S-L-connected (S-M-connected) subset of X.

Theorem: 4.17

Every S-R-connected space is connected.

Proof:

Let Y be an S-R-connected space. Suppose X is not connected. Then Y=A$\cup$B is a separation then it is a S-R-separation. Since every open set is S-R-open set in Y. This is contradiction to fact that (Y,σ) is S-R-connected. Therefore hence (Y,σ) is connected.

Theorem: 4.18

Every S-S-connected space is connected.

Proof:

Let Y be an S-S-connected space. Suppose X is not connected. Then Y=A$\bigcup$B is a separation then it is a S-S-separation. Since every closed set is S-S-closed set in Y. This is contradiction to fact that (Y, σ) is S-S-connected. Therefore hence (Y, σ) is connected.

Definition: 4.19

Let Y be a subset of X. Then Y=A$\bigcup$B is said to be a S-R-separation (S-S-separation) of Y if A and B are non empty disjoint S-R-open (S-S-closed) sets in X. If there is no S-R-separation (S-S-separation) of Y then Y is said to be S-R-connected (S-S-connected) subset of X.

Theorem: 4.20

Let(X,τ) be a finitely S-M-additive topological space and X=A$\bigcup$B be a S-L-separation of X. if Y is S-L-open and S-L-connected subset of X, then Y is completely contained in either A or B.

Proof:

X=A$\bigcup$B is a S-L-separation of X. Suppose Y intersects both A and B then $Y=(A\bigcap Y)\bigcup(B\bigcap Y)$ is a S-L-separation of Y, Since X is finitely S-M-additive. This is a contradiction.

Theorem: 4.21

Let(X,τ) and (Y, σ) be bijection. Then

1) f is S-L-continuous(S-M-continuous) and X is S-L-connected(S-M-connected) $\Rightarrow$Y is connected.
2) f is continuous and X is S-L-connected(S-M-connected) $\Rightarrow$Y is connected.
3) f is S-L-open(S-M-closed) and Y is S-L-connected(S-M-connected)$\Rightarrow$ X is connected.
4) f is open then X is connected$\Rightarrow$Y is S-L-connected(S-L-connected).
5) f is P-L-irresolute(S-M-irresolute) then X is S-L-connected(S-M-connected)$\Rightarrow$Y is S-L-connected (S-M-connected).
6) f is S-R-resolute(S-S-resolute) then Y is S-R-connected(S-S-connected)$\Rightarrow$X is S-R-connected(S-S-connected).Proof:1)Suppose Y= A$\bigcup$B is a separation of Y, then $X=f^{-1}(Y)=f^{-1}(A)\bigcup f^{-1}(B)$ is a S-L-separation of X which is contradiction. Therefore Y is connected.Proof for (2) to (7) is similar to the above proof.

Theorem: 4.22

A topological space (X, τ) is S-L-disconnected if and only if there is exists aS-L-continuous map of X onto discrete two point space Y = {0, 1}.

Proof:

Let (X, τ) be S-L-disconnected and Y = {0, 1} is a space with discrete topology. X = A$\cup$B be a S-L-separation of X. Define f:X$\rightarrow$Y such that f(A) = 0 and f(B) = 1. Obviously f is onto, S-L-continuous map. Conversely, let f: X$\rightarrow$Y beS-L-continuous onto map. Then X = $f^{-1}(0)\cup f^{-1}(1)$ is a S-L-separation of X.

Theorem: 4.23

Let (X, τ) be a finitely S-M-additive topological space. If {A_α } is an arbitrary family of S-L-open S-L-connected subset of X with the common point p then $\cup A_\alpha$ is S-L-connected

Proof:

Let $\cup A_\alpha$ = B$\cup$C be aS-L-separation of $\cup A_\alpha$. Then B and C are disjoint non empty S-L-open sets in X.P$\in \cup A_\alpha$ $\Rightarrow$P$\in$B or P$\in$C. Assume that P$\in$B. Then by theorem (4.20)A_α is completely contained in B for all α (since P$\in$B). Therefore C is empty which is contradiction.

Corollary: 4.24

Let (X, τ) be a finitely S-M-additive topological space. If {A_n} is a sequence ofS-L-open S-L-connected subsets of X such that $A_n\cap A_{n+1}\neq\Phi$, for all n. Then $\cup A_n$ is S-L-connected

Proof:

This can be proved by induction on n. By theorem (4.23), the result is true for n=2. Assume that the result to be true when n=k. Now to prove the result when n=k+1. By the hypothesis $\bigcup_{i=1}^{n} A_i$ is S-L-connected. Now ($\bigcup_{i=1}^{n} A_i$)$\bigcap$ A $_{k+1} \neq \Phi$. Therefore by theorem (4.23) $\bigcup_{i=1}^{k+1} A_i$ is S-L-connected. By induction hypothesis the result is true for all n.

Corollary: 4.25

Let (X, τ) be a finitely S-M-additive topological space. Let $\{A_\alpha\}_{\alpha \in \Omega}$ be an arbitrary collection ofS-L-open S-L-connected subsets of X. Let A be aS-L-openS-L-connected subsets of X. If $A \cap A_\alpha \neq \Phi$ for all α then $A \cup (\cup A_\alpha)$ is S-L-connected.

Proof:

Suppose that $A \cup (\cup A_\alpha)$ =B$\cup$C be a S-L-separation of the subset $A \cup (\cup A_\alpha)$ since A$\subseteq$ B$\cup$C by theorem (4.23), A$\subseteq$B or A$\subseteq$C. Without loss of generality assume that A$\subseteq$B. Let $\alpha \in \Omega$ be arbitrary. $A_\alpha \subseteq$B$\cup$C,by theorem (4.20), $A_\alpha \subseteq$B or A$\subseteq$C. But$A \cap A_\alpha \neq \Phi \Rightarrow A_\alpha \subseteq$ B. Since α is arbitrary, $A_\alpha \subseteq$ B for all α. Therefore $A \cup (\cup A_\alpha) \subseteq$B which implies C=$\Phi$. Which is a contradiction. Therefore $A \cup (\cup A_\alpha)$ is S-L-connected.

Definition: 4.26

A space (X, τ) is said to be totally S-L-disconnected if its only S-L-connected subsets are one point sets.

Example: 4.27

Let (X, τ) be an indiscrete topological space with more than one point. Here all subsets are S-L-open. If A={X_1,X_2} then A={X_1}$\cup${X_2} is S-L- separation of A. Therefore any subset with more than one point is S-L-disconnected. Hence (X, τ) is totally S-L-disconnected

Remark: 4.28

Totally S-L-disconnectedness implies S-L-disconnectedness.

Definition: 4.29

In a topological space (X, τ) a point x$\in$X is said to be in S-L-boundary of A, (S-L-Bd (A)) if every S-L-open set containing x intersects both A and X-A.

Theorem: 4.30

Let (X, τ) be a finitely S-M-additive topological space and let A be a subset of X. If C is S-L-open S-L-connected subset of X that intersects both A and X-A then C intersects S-L-Bd(A).

Proof:

It is given that $C \cap A \neq \Phi$and $C \cap A^c \neq \Phi$. Now C= (C$\cap$A) $\cup$ (C$\cap A^c$) is a non empty disjoint union.Suppose both are S-L-open then it is a contradiction to the fact that C is S-L-connected. Hence either C$\cap$A or C$\cap A^c$ is not S-L-open. Suppose that C$\cap$A is not S-L-open. Then there exists x $\in$ C$\cap$A which is not a S-L-interior point of C$\cap$A. Let U be a S-L-open set containing x. Then U$\cap$C is a S-L-open set containing x. Hence $(U \cap C) \cap (C \cap A^c) \neq \Phi$. This implies U intersects both A and A^c and therefore x$\in$S-L-Bd (A). Hence C$\cap$S-L-Bd(A)$\neq \Phi$.

Theorem: 4.31 (Generalisation of intermediate Value theorem)

Let f:X$\rightarrow$R be a S-L-continuous map where X is a S-L-connected space and R with usual topology. If x, y are two points of X, a=f(x) and b=f(y) then every real number r between a and b is attained at a point in X.

Proof:

Assume the hypothesis of the theorem. Suppose there is no point $c\in X$ such that f(c)=r. Then A=(-∞,r) and B=(r,∞) are disjoint open sets in R, since f is S-L-continuous, f^{-1} (B) are S-L-open in X. X = f^{-1}(A) $\cup$ f^{-1}(B) which is aS-L-separation of X. This is a contradiction to the fact that X isS-L-connected. Therefore there exists$c\in X$ such that f(c)=r.

Definition 4.32

Let X be a topological space and $A\subseteq X$. The S-M-closure of A is defined as theintersection of all S-M-closed sets in X containing A, and is denoted byS-M-(cl(A)). It is clear that S-M-(cl(A)) is S-M-closed set for any subset A of X .

Proposition 4.33

Let X be a topological space and $A\subseteq B\subseteq X$.

Then:

i. S-M-(cl(A))$\subseteq$S-M-(cl(B))
ii. $A\subseteq$S-M-(cl(A))
iii. A is S-M-closed iff$A$= S-M-(cl(A))

Definition 4.34

Let X be a topological space and $x\in X, A\subseteq X$. The Point x is called a S-L-limit point of A if each S-L-open set containing x , contains a point of A distinct from x.

We shall call the set of all S-L-limit points of A the S-L-derived set of A and denoted it by S-L-(A'). Therefor e$x\in$S-L-(A')iff for every S-L-open set G in X such that $x\in G$ implies that $(G\cap A)-\{x\}\neq\phi$.

Proposition 4.35

Let X be a topological space and $A\subseteq B\subseteq X$.

Then:

S-M-(cl(A))= $A\cup$ P-L-(A')

A is S-M-closed set iffS-L-(A')$\subseteq A$

S-L-(A')$\subseteq$S-L-(B')

S-L-(A')$\subseteq A'$ (v)S-M-(cl(A))$\subseteq cl(A)$

Proof

(i) If $x\notin$S-M-(cl(A)) , then there exists a S-M-closed set F in X such that $A\subseteq F$ and $x\notin F$. Hence $G= X-F$ is aS-L-open set such that $x\in G$ and $G\cap A=\phi$.

Therefore $x\notin A$ and $x\notin$S-L-(A'), then $x\notin A\cup$ S-L-(A'). Thus $A\cup$ S-L-(A')$\subseteq$S-M-($cl(A)$. On the other hand, $x\notin A\cup$ S-L-(A')implies that there exists a S-L-open set G in X such that $x\in G$ and $G\cap A=\phi$. Hence F = $X-G$ is aS-M-closed set in X such that $A\subseteq F$ and $x\notin F$. Hence $x\notin$S-M-($cl(A)$). Thus S-M-($cl(A)$)$\subseteq A\cup$ S-L-(A'). Therefore S-M-($cl(A)$)$\subseteq A\cup$ S-L-(A').

For (ii), (iii), (iv) and (v) the proof is similar.

Definition 4.36

Let X be a topological space. Two non- empty subsets A and B of X are Called S-M-separated iff S-M-($cl(A)$)$\cap B = A\cap$ S-M-($cl(B)$)$=\phi$.

Remark 4.37

A set A is called S-L-M-clopeniff it is S-L-open and S-M-closed.

Theorem 4.38

Let X be a topological space, then the following statements are equivalent:

(i) X is a S-M-connected space.

(ii) X cannot be expressed as the union of two disjoint, non- empty and S-M-closed sets.

(iii) The only S-L-M-clopen sets in the space are X and ϕ.

Example 4.39

In this example we show that S-M-connectivity is not a hereditary property. Let $X = \{a,b,c,d\}$ and $Tx=\{\{a\},\{a,b\},\{a,c\},\{a,b,c\},\phi, X\}$ be a topology on X.

The S-L-open sets are: $\{a\},\{a,b\},\{a,c\},\{a,b,c\},\{a,d\},\{a,b,d\},\{a,c,d\},\phi, X$. It isclear that X isS-L-connected space since the only S-L-clopen sets are X and ϕ. Let$Y = \{b,c\}$, then $T_y=\{\{b\},\{c\},Y,\phi\}$. It is clear that Y is not S-L-connected space since$\{b\} \neq \phi$, $\{b\} \neq Y$ and $\{b\}$ is S-L-M-clopenset in Y. Thus aS-M-connectivity is not a hereditary property.

Proposition 4.40

Let A be aS-M-connected set and H, K are S-M-separated sets. If $A \subseteq H \cup K$ then

Either $A \subseteq H$ or $A \subseteq K$.

Proof

Suppose A is S-M-connected set and H, K are S-M-separated sets such that $A \subseteq H \cup K$. Let $A \not\subset H$ or $A \not\subset K$. Suppose $A_1 = H \cap A = \phi$ and $A_2 = K \cap A \neq \phi$. Then $A = A_1 \cup A_2$. Since $A_1 \subseteq H$, henceS-M-$(cl(A_1)) \subseteq$S-M-$(cl(H))$. Since S-M-$(cl(H)) \cap K = \phi$, then S-M-$(cl(A_1)) \cap A_2 = \phi$. Since $A_2 \subseteq K$, hence S-M-$(cl(A_2)) \subseteq$S-M-$(cl(K))$.

Since S-M-$(cl(K)) \cap H = \phi$ then S-M-$(cl(A_2)) \cap A_1 = \phi$But $A = A_1 \cup A_2$, therefore A is not S-M-connectedspace which is a contradiction. Then either $A \subseteq H$ or $A \subseteq K$.

Proposition 4.41

If H is S-M-connected set and $H \subseteq E \subseteq$P-M-$(cl(H))$ then E is S-M-connected.

Proof

If E is not S-M-connected, then there exists two sets A, B such that S-M-$(cl(A)) \cap B = A \cap$S-M-$(cl(B)) = \phi$ and $E = A \cup B$. Since $H \subseteq E$,thus either $H \subseteq A$ or$H \subseteq B$. Suppose $H \subseteq A$, then S-M-$(cl(H)) \subseteq$S-M-$(cl(A))$, thus S-M-$(cl(H)) \cap B =$S-M-$(cl(A)) \cap B = \phi$. But $B \subseteq E \subseteq$S-M-$(cl(H))$, thus S-M-$(cl(H)) \cap B = B$. Therefore $B = \phi$which is a contradiction. Thus E is S-M-connected set. If $H \subseteq E$, then by the same way we can prove that $A = \phi$which is a contradiction. Then E is S-M-connected.

Corollary 4.42

If a space X contains a S-M-connected subspace A such that S-M-$(cl(A))=X$, then X is S-M-connected.

Proof

Suppose A is a S-M-connected subspace of X such that S-M-$(cl(A))=X$. Since $A \subseteq X =$S-M-$(cl(A))$, then by proposition 4.41, X is S-M-connected.

Proposition 4.43

If A is S-M-connected set then S-M-$(cl(A))$ is S-M-connected.

Proof: (4.43)

Suppose A is S-M-connected set and S-M-$(cl(A))$ is not. Then there exist two S-M-separated Sets H, K such that S-M-$(cl(A))=H \cup K$. But $A \subseteq$S-M-$(cl(A))$,, then $A \subseteq H \cup K$ and since A is S-M-connected set then either $A \subseteq H$ or $A \subseteq K$ (by proposition 4.40)

1) If $A \subseteq H$, then S-M-$(cl(A)) \subseteq$S-M-$(cl(H))$. But S-M-$(cl(H)) \cap K = \phi$, hence S-M-$(cl(A)) \cap K = \phi$. Since $K \subseteq$S-M-$(cl(A))$, then $K = \phi$which is a contradiction.

2) If $A \subseteq K$, then the same way we can prove that $H = \phi$ which is a contradiction.

ThereforeS-M-$(cl(A))$ is S-M-connected set.

Proposition 4.44

Let X be a topological space such that any two elements a and b of X are contained in some S-M-connected subspace of X. Then X is S-M-connected.

Proof

Suppose X is not S-M-connected space. Then X is the union of two S-M-separated Sets A *and* B. Since A, B are non-empty sets, thus there exist a, b such that $a \in A$, $b \in B$. Let H be aS-M-connected subspace of X which contains a and b. Therefore by proposition 4.40 either $H \subseteq A$ or $H \subseteq B$ which is a contradiction since $A \cap B = \phi$.

Then X is S-M-connected space.

Proposition 4.45

If A and B are S-L-connected subspace of a space X such that $A \cap B \neq \phi$, then $A \cup B$ is S-L-connected subspace.

Proof

Suppose that $A \cup B$ is not S-L-connected. Then there exist two S-L-separated sets H and K such that $A \cup B = H \cup K$. Since $A \subseteq A \cup B = H \cup K$ and A isS-L-connected, then either $A \subseteq H$ or $A \subseteq K$. Since $B \subseteq A \cup B = H \cup K$ and B is S-L-connected, then either $B \subseteq H$ or $B \subseteq K$.

(1) If $A \subseteq H$ and $B \subseteq H$, then $A \cup B \subseteq H$. Hence $K = \phi$ which is a contradiction.

(2) If $A \subseteq H$ and $B \subseteq K$, then $A \cap B \subseteq H \cap K = \phi$. Therefore $A \cap B = \phi$ which is a contradiction.

By the same way we can get a contradiction if $A \subseteq K$ and $B \subseteq H$ or if $A \subseteq K$ and $B \subseteq K$. Therefore $A \cup B$ is S-L-connected subspace of a space X.

Theorem 4.46

If X and Y are S-L-connected spaces, then $X \times Y$ is S-L-connected space.

Proof

For any points (x_1, y_1) and $(x_2, y_2)$of the space $X \times Y$, the subspace $X \times \{y_1\} \cup \{x_2\} \times Y$ contains the two points and this subspace is S-L-connected, since it is the union of two S-L-connected subspaces with a point in common (by proposition 4.45).

Thus $X \times Y$ is S-L-connected (by proposition 4.44).

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