IJETST- Vol.||03||Issue||03||Pages 3638-3646||March||ISSN 2348-9480 2016

Open access Journal International Journal of Emerging Trends in Science and Technology
Impact Factor: 2.838 DOI: http://dx.doi.org/10.18535/ijetst/v3i03.04

Irredundant Complete Domination Number of Graphs

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Abstract

The concept of complete graphs with real life application was introduced in [17] .In [14], A. Nellai Murugan et.al., was introduced the concept of complete dominating number of a graph. In this paper, We introduce a new domination parameter called Irredundant complete dominating set of K_4 -e , A subset S of V of a non trivial graph G is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating set in G. A subset S of V of a nontrivial graph G is said to be complete dominating set, If for each $x \in V, N[x] \cap N[S-\{x\}] = V - S$ denoted by S' is the complete dominating set. The minimum cardinality taken over all complete dominating set is called the complete domination number and is denoted by $\gamma_n(G)$[14].A set $x \in S$ is said to be redundant in S if $N[x] \subseteq N[S-\{x\}]$ othewise x is said to be irredundant in S . Finally , S is called an irredundant set if all $x \in S$ are irredundant in S , Otherwise S is a redundant set .A subset S of v of a nontrivial graph G is said to be an Irredundant complete dominating set if S is an irredundant and complete . The minimum cardinality taken over all an irredundant complete dominating set is called an Irredundant complete domination number and is denoted by $\gamma_{irn}(G)$.

Mathematics Subject Classification: 05C69

Keywords: *Complete dominating set, Complete domination number, Irredundant Complete dominating set, Irredundant Complete domination number*

1 Introduction

The concept of domination in graphs evolved from a chess board problem known as the Queen problem- to find the minimum number of queens needed on an 8x8 chess board such that each square is either occupied or attacked by a queen. C.Berge [3] in 1958 and 1962 and O.Ore [8] in 1962 started the formal study on the theory of dominating sets. Thereafter several studies have been dedicated in obtaining variations of the concept. The authors in [7] listed over 1200 papers related to domination in graphs in over 75 variation.

Throughout this paper , G(V , E) a finite , simple , connected and undirected graph where V denotes its vertex set and E its edge set. Unless otherwise stated the graph G has n vertices and m edges. Degree of a vertex v is denoted by d(v), the *maximum degree* of a graph G is denoted by Δ(G). Let C_n a cycle on n vertices, P_n a path on n vertices by and a complete graph on n vertices by K_n. A graph is *connected* if any two vertices are connected by a path . A maximal connected subgraph of a graph G is called a *component* of G .The *number of components* of G is denoted by ω(G).

The *complement* $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G. A tree is a connected acyclic graph. A *bipartite graph* is a graph whose vertex set can be divided into two disjoint sets V_1 and another in V_2 . A *complete bipartite graph* is a bipartite graph with partitions of order. $|V_1|$ =m and $|V_2|$=n, is denoted by $K_{m,n}$. A star denoted by $K_{1,n-1}$ is a *tree* with one root vertex and n-1 pendant vertices. A bistar, denoted by B(m,n) is the graph obtained by joining the root vertices of the stars denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A *wheel* graph denoted by W_n is a graph with n vertices formed by connecting a single vertex to all vertices of $C_{n-1.}$ A *Helm* graph denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex to each vertex in the outer cycle of W_n.

The *chromatic number* of a graph G denoted byχ(G) is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colours . For any real number x , $\lceil x \rceil$ denotes the largest integer greater than or equal to x and $\lfloor x \rfloor$ the smallest integer less than or equal to x. A Nordhaus- Gaddum – type result is a lower or upper bound on the sum or product of a parameter of a graph and its complement. Throughout this paper, we only consider undirected graphs with no loops .The basic definitions and concepts used in this study are adopted from[11].

Given a graph G = (V(G) , E(G)) , the cardinality $|V(G)|$ of the vertex set V(G) is the order of G is n. The distance d_G(u ,v) between two vertices u and v of G is the length of the *shortest path* joining u and v .If d_G(u ,v) = 1, u and v are said to be adjacent.

For a given vertex v of a graph G , The open neighbourhood of v in G is the set N_G(v) of all vertices of G that are adjacent to v.

The degree deg_G(v) of v refers to$|N_G(v)|$, and $\Delta(G) = \max\{deg_G(v)$: v ϵ $V(G)\}$. The closed neighbourhood of v is the set $N_G[v] = N_G(v) \cup v$ for S ⊆ $V(G)$, $N_G(S) = \cup_{v \epsilon S} N_G(v)$ and $N_G[v]$= $N_G(S) \cup S$. If $N_G[v]$==V(G), then S is a dominating set in G. The minimum cardinality among dominating sets in G is called the *domination number* of G and is denoted by $\gamma(G)$.

A dominating set S in a graph G is an independent dominating set if for every pair of distinct vertices u and v in S, u and v are non adjacent in G . The minimum cardinality $\gamma_i(G)$ of an independent dominating set in G is called the *independent domination number* of G.

2. Relationships between domination and irredundant complete domination numbers:

2. Main Results

Definition:2.0:A subset S of v of a nontrivial graph G is said to be an Irredundant complete dominating set if S is an irredundant and complete . The minimum cardinality taken over all an irredundant complete dominating set is called an Irredundant complete domination number and is denoted by $\gamma_{irn}(G)$.

Example2.1 : For any graph G=C_4 is an irredundant complete dominating set if S={u,v}, since N[u]={u,w,x},N[v]={v,w,x}, N[u]-N[S-u]={u} $\neq \emptyset$ and N[u] ∩ N[S-u]={x,w}=V-S. Hence S is an irredundant complete dominating set of G with $\gamma_{irn}(G)$=n-(n-2)=2 by fig.1. Since $|N_G[v]| = |N_G(S) \cup S| = (n-2)+2 = n$ that is, $|N_G[v]| = |S'| \cup |S|$=(n-2)+2=n.

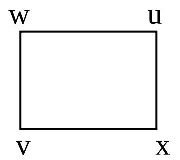

Figure 1: $\boldsymbol{\gamma_{irn}(G) = [n-(n-2)]}$**=2**

Example2.2: For any graph G=C_4 +e or K_4-e is an irredundant complete dominating set if S={w,x}, since N[w]={u,v,w},N[x]={u,v,x}, N[w]-N[S-w]={w} ≠ ∅ and N[w] ∩ N[S-w]={u,v}=V-S. Hence S is an irredundant complete dominating set of G with $\gamma_{irn}(G)$=n-(n-2)=2 by fig.2.

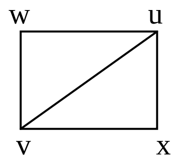

Figure 2: $\boldsymbol{\gamma_{irn}(G) = [n-(n-2)]=2}$

Example2.3 : For any graph G=K_4 is not an irredundant complete dominating set if S={u,v}, since N[u]={u,v,w,x},N[v]={u,v,w,x}, Here N[u]-N[S-u]={u}= ∅ and N[u] ∩N[S-u]={u,v,x,w}≠V-S. Hence S is not an irredundant complete dominating set of G by fig.3.

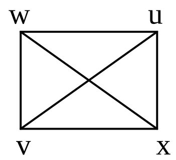

Figure 3: It is not an irredundant complete dominating set

Example2.4 : For any graph G=K_4 –e is not an irredundant complete dominating set where uv∈E. If S={u,v}, since N[u]={u,v,u_i, v_i} ,N[v]={u,v,u_i, v_i}, Here N[u]-N[S-u]={u}= ∅ and N[u] ∩N[S-u]={u,v, u_i, v_i}≠V-S. Hence S is not an irredundant complete dominating set of G by fig.4.

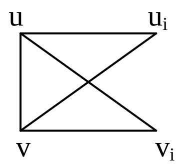

Figure 4: It is not an irredundant complete dominating set

Result2.5: For any graph G=C_4 +v is an irredundant complete dominating set if S={u,v}, since N[u]={u,w, u_i, v_i },N[v]={v,w, u_i, v_i }, N[u]-N[S-u]={u} ≠ ∅ and N[u] ∩ N[S-u]={ u_i, v_i ,w}=V-S. Hence S is an irredundant complete dominating set of G with $\gamma_{irn}(G)$=n-(n-2)=2.

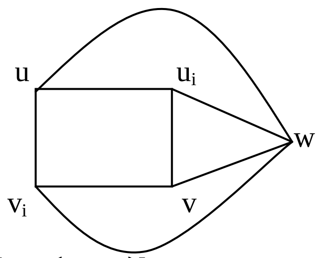

Figure 5: $\boldsymbol{\gamma_{irn}(G) = [n-(n-2)]=2}$

Result2.6: For any graph G=$H_1 + H_2$ where H_1 and H_2 be the sub graph of G and if either H_1 or H_2 must be the irredundant complete dominating set .

Result 2.7: For any graph G=$H_1 + \bigcup_{i=2}^{n} H_i$ .If H_1 has the irredundant complete domination number then the graph G=$H_1 + \bigcup_{i=2}^{n} H_i$ has an irredundant complete domination number.

Result 2.8: For any graph G=$[C_4 + \bigcup_{i=1}^{4} H_i] - 4e$ .If each H_i is a singleton vetex then C_4 has the irredundant complete domination number then the graph G=$[C_4 + \bigcup_{i=1}^{4} H_i] - 4e$ has an irredundant complete domination number with $\gamma_{irn}(G)$=2n-(2n-2) since $|G|$=$|C_4| + |\bigcup_{i=1}^{4} H_i|$=4+4=8 where each H_i is of order 1.

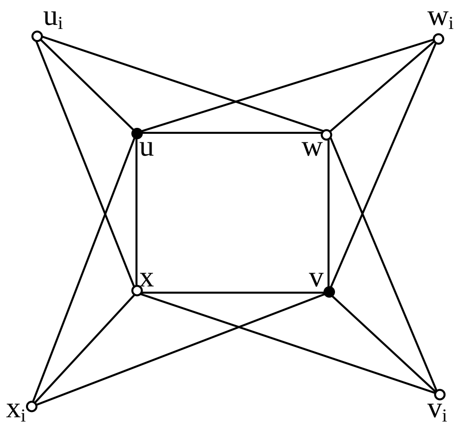

Figure 6: $\boldsymbol{\gamma_{irn}(G) = [2n - (2n-2)]}$**=2**

Result 2.9: For any graph G=K_5 -2e has an irredundant complete dominating set of G if S=$\{u_1, u_2\}$

Since N$[u_1] = \{u_1, v_1, v_2, v_3\}$, N$[u_2] = \{u_2, v_1, v_2, v_3\}$, N$[u_1]$ − N$[u_2] \neq \emptyset$ and N$[u_1] \cap$ N$[u_2]$=$\{ v_1, v_2, v_3\} = V - S$ and its $\gamma_{irn}(G)$=n-(n-2)=2.

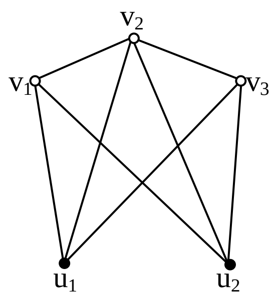

Figure 7: $\boldsymbol{\gamma_{irn}(G) = [n - (n-2)]}$**=2**

Result 2.10: For any graph G=K_5 -3e has an irredundant complete dominating set of G if S=$\{u_1, u_2, u_3\}$

Since N$[u_1] = \{u_1, v_1, v_2\}$, N$[u_2] = \{u_2, v_1, v_2\}$, N$[u_3] = \{u_3, v_1, v_2\}$, N$[u_1]$ − N$[u_2] \neq \emptyset$ and N$[u_1] \cap$ N$[u_2]$=$\{ v_1, v_2\} = V - S$ and its $\gamma_{irn}(G)$=n-(n-3)=3.

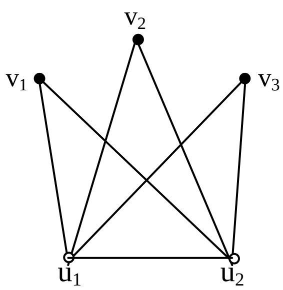

Figure 8: $\boldsymbol{\gamma_{irn}(G) = [n - (n-3)]}$**=3**

Result 2.11: For any graph G=K_6 -3e has an irredundant complete dominating set of G if S=$\{u_1, u_2, u_3\}$, Since N$[u_1] = \{u_1, v_1, v_2, v_3\}$, N$[u_2] = \{u_2, v_1, v_2, v_3\}$, N$[u_3] = \{u_3, v_1, v_2, v_3\}$, N$[u_1]$ − N$[u_2] \neq \emptyset$ and N$[u_1] \cap$ N$[u_2]$=$\{ v_1, v_2, v_3\} = V - S$ and its $\gamma_{irn}(G)$=n-(n-3)=3.

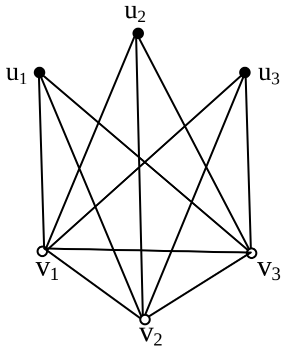

Figure 9: $\boldsymbol{\gamma_{irn}(G) = [n - (n - 3)]}$**=3**

Result 2.12: For any graph G=K_6 -e has an irredundant complete dominating set of G if S={u_1, u_2}

Since N[u_1] = {u_1, v_1, v_2, v_3, v_4 }, N[u_2] = {u_2, v_1, v_2, v_3 , v_4 }, N[u_1] − N[u_2] ≠ ∅ and N[u_1] ∩ N[u_2]={ v_1, v_2, v_3, v_4} = $V - S$ and its $\gamma_{irn}(G)$=n-(n-2)=2.

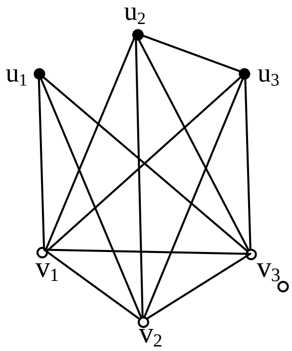

Figure 10: $\boldsymbol{\gamma_{irn}(G) = [n - (n - 2)]}$**=2**

Result 2.13: For any graph G= [K_6 –e]+H where H is a subgraph of G and its $\gamma_{irn}(G)$=n-(n-2)=2.

Result 2.14: For any graph G= [K_6 –3e]+$\cup_{i=1}^{n} H_i$ where H_i are subgraphs of G and its $\gamma_{irn}(G)$=n-(n-3)=3.

Lemma 2.15: For any graph G . Let S be an irredundant complete dominating set of G. If there exists vertices u and v such that N[u]-N[S-u]≠ ∅ and N[u] ∩N[S-u]=V-S.

Proof: For any graph G. Let S={u,v} such that since N[u]={u,u_i, v_i} ,N[v]={v,u_i, v_i} where 'i' is the neighbour of the respective vertex we have N[u]-N[S-u]≠ ∅ and N[u] ∩N[S-u]= {u_i, v_i}= V-S.

Corollary 2.16: For any graph G has an irredundant complete dominating set if there exists $u_i, v_i \in V - S$ such that $\cap N(V - S) = S$.

Proof: By the definition of irredundant complete dominating set.

Corollary 2.17: For any graph G=$H_1 + H_2$.Let S is a irredundant complete dominating set of H_1 or H_2. If there exists vertices u and v such that uv$\notin E$ we have N[u]-N[S-u]≠ ∅ and N[u] ∩N[S-u]=V-S

Proof: By the definition of irredundant complete dominating set.

Lemma 2.18: For any graph G . let S be an irredundant complete dominating set of G.If there exists vertices u and v such that uv$\notin E$ we have N[u]-N[S-u]≠ ∅ and N[u] ∩N[S-u]=V-S and $\cup N(V - S) = V$.

Proof: For any graph G given S be an irredundant complete dominating set of G.If there exists vertices u and v such that uv$\notin E$ we have N[u]={u,u_i, v_i} ,N[v]={v,u_i, v_i} and $\bigcap N[S] = V - S$

also N(u_i) ={ v_i, u, v}, N(v_i) = {u_i, u, v} which implies that $\bigcup N(V-S) = \{u, v, u_i, v_i\} = V$.

Corollary 2.19: For any graph G=$H_1 + H_2$.Let S is a irredundant complete dominating set of H_1 of H_2 .If there exists vertices u and v such that uv$\notin E$ we have N[u]-N[S-u]$\neq \emptyset$ and N[u] ∩N[S-u]=V-S and $\bigcup N(V-S) = V$.

Proof: By the definition of irredundant complete dominating set.

Result 2.20: For any Corona graph G∘H of a graph G and H has not an irredundant complete dominating set , since if S={u,v,u_3, u_5} then N[u]={u,u_1, u_6, u_2}, N[v]={v,u_1, u_6, u_4}, N[u_6]={u_1, u_3}, N[u_5]={u_5, u_6}, since N[u] ∩N[S-u] ≠V-S it is not a complete dominating set . Hence f.or any Corona graph G∘Hof a graph G and H has not an irredundant complete dominating set.

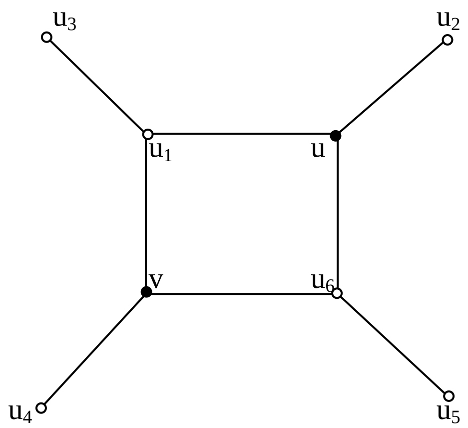

Figure 11: It is not an irredundant complete dominating set

Theorem 2.21: For any graph G . Let S be any irredundant complete dominating set in a graph G then their domination numbers are (i) $\gamma_{irn}(G)$=n-(n-2)=2 if G≅K_4 –e or K_3 –e (ii) $\gamma_{irn}(G)$=n-(n-2)=2 if G≅C_4+H(or) G ≅C_4 (or) G ≅C_4+$\bigcup_i^n H_i$ (iii) $\gamma_{irn}(G)$=n-(n-2)=2 if G≅K_5-e (or) G ≅ (K_5-e)+H(Or) G ≅ (K_5-e)+$\bigcup_i^n H_i$ (iv) $\gamma_{irn}(G)$=n-(n-3)=3 if G≅K_6-3e (or) G ≅ (K_5-e)+H(or) G ≅ (K_5-3e)+$\bigcup_i^n H_i$ (or) G ≅ (K_6-3e)+$\bigcup_i^n H_i$

Proof: For any graph G, Given S and S' be any irredundant complete dominating set of a graph G, Since there exists vertices u,v∈S such that N[u]-N[S-u]$\neq \emptyset$ and N[u] ∩N[S-u]=V-S =S' and $|N_G[v]| = |N_G(S)| \cup |S| = (n-2) + 2 = n$ and it is by the definition of irredundant complete dominating set of G , we have $N_G[S] = S'$ with uv$\notin E$

Case (i) For any graph G=C_4 +e or K_4-e is an irredundant complete dominating set if S={w,x}, since N[w]={u,v,w},N[x]={u,v,x}, N[w]-N[S-w]={w} $\neq \emptyset$ and N[w] ∩N[S-w]={u,v}=V-S. Hence S is an irredundant complete dominating set of G with $\gamma_{irn}(G)$=n-(n-2)=2. Since from the above result $|N_G[v]| = |N_G(S)| \cup |S| = (n-2) + 2 = n$,$\forall v \in G$.

Case (ii) For any graph G≅C_4+H (or) G ≅C_4 (or) G ≅C_4+$\bigcup_i^n H_i$ has a irredundant complete dominating set if S={u,v} and uv$\notin E$, since N[u]={u, u_i, v_i },N[v]={v, u_i, v_i }, N[u]-N[S-u]={u}$\neq \emptyset$ and N[u] ∩N[S-u]={ u_i, v_i }=V-S and $|V-S|$=2, Since it satisfies the conditions of irredundancy and and complete domination , S is irredundant complete dominating set of G and from the above result $|N_G[v]| = |N_G(S)| \cup |S| = (n-2) + 2 = n$,$\forall v \in G$.We have $\gamma_{irn}(G)$=n-(n-2)=2.

Subcase(a) Suppose G ≅C_4 +H, C_4 has an irredundant complete dominating set by case (ii) we have by adding subgraph H of G each vertex of H is adjacent with S. Here S is the irredundant complete dominating set of G and its $\gamma_{irn}(G)$=n-(n-2)=2.

Subcase(b) Suppose $G \cong C_4 + \cup_i^n H_i$ then C_4 has an irredundant complete dominating set by subcase (a)case (ii) C_4+H has an irredundant complete dominating set. Also by taking union of all H_i, i=1,2... then every vertex of H_i is adjacent with S. Hence S is the irredundantbcomplete dominating set of G and its $\gamma_{irn}(G)$=n-(n-2)=2.

Case(iii) G≅K_5-e (or) $G \cong$ (K_5-e)+H(Or) $G \cong$ (K_5-e)+$\cup_i^n H_i$ has an irredundant complete dominating set if S={u,v} and uv$\notin E$ }, since N[u]={u, x, y, z},N[v]={v, x, y,z}, N[u]-N[S-u]={u}$\neq \emptyset$ and N[u] ∩N[S-u]={ x, y, z}=V-S and $|V-S|$=2, Since it satisfies the conditions of irredundancy and and complete domination , S is irredundant complete dominating set of G and from the above result $|N_G[v]| = |N_G(S)| \cup |S| = (n-2)+2 = n$,$\forall v \in G$.We have $\gamma_{irn}(G)$=n-(n-2)=2.

Subcase(a) For any graph G≅K_5-2e (or) G≅K_5-3e (or) G≅K_5-4e has an irredundant complete dominating set as from the case (iii),we have N[u]-N[S-u]={u}$\neq \emptyset$ and N[u] ∩N[S-u]={ x, y, z}=V-S and $|V-S|$=2, Since it satisfies the conditions of irredundancy and and complete domination , S is irredundant complete dominating set of G and $\gamma_{irn}(G)$=n-(n-2)=2.

Subcase(b) For any graph $G \cong$ (K_5-e)+ $\cup_i^n H_i$ (or) $G \cong$ (K_5-2e)+ $\cup_i^n H_i$ (or) $G \cong$ (K_5-3e)+ $\cup_i^n H_i$ (or) $G \cong$ (K_5-4e)+$\cup_i^n H_i$.The proof is by Subcase(a) of case (ii) we have the $\gamma_{irn}(G)$=n-(n-2)=2.

Case(iv) For any graph $G \cong$ (K_6-3e)+$\cup_i^n H_i$ (or) $G \cong$ (K_6-3e) has an irredundant complete dominating set if S={u,v,w} and uvw$\notin E$ }, since N[u]={u, x, y, z},N[v]={v, x, y,z}, N[w]={w, x, y,z} and N[u]-N[S-u]={u}$\neq \emptyset$ also N[u] ∩N[S-u]={ x, y, z}=V-S and $|V-S|$=3 Since it satisfies the conditions of irredundancy and and complete domination , S is irredundant complete dominating set of G and $\gamma_{irn}(G)$=n-(n-3)=3.

Theorem 2.22: Every irredundant complete dominating set is a degree equitable dominating set.

Proof: Given S is an irredundant complete dominating set if S={u,v} and uv $\notin E$, since N[u]={u, u_i, v_i },N[v]={v, u_i, v_i }, N[u]-N[S-u]={u} $\neq \emptyset$ and N[u] ∩ N[S-u]={ u_i, v_i }=V-S, Hence S={u,v} for i=1,2...n and N(u_i) ={ u, v}, N(v_i) = {u, v} which implies that {N(u_i) ∩ N(v_i)}={u,v}=s=[n-(n-2)],HereN(u)=N(v) by $|N(u)| = |N(v)|$ where N(u)=n and N(v)=n which give $\frac{|N(u)|}{|N(v)|} = 1$,which implies that $|d(u) - d(v)| = 0$. By the definition of degree equitable dominating set the given irredundant complete dominating set is a degree equitable dominating set.

Theorem 2.23: Every irredundant complete dominating set is an independent dominating set with uv$\notin E$

Proof: S is an irredundant complete dominating set if S={u,v} , since N[u]={u, u_i, v_i},N[v]={v, u_i, v_i}, N[u]-N[S-u]={u}$\neq \emptyset$ and N[u] ∩N[S-u]={ u_i, v_i}=V-S, Hence S={u,v} for i=1,2...n and N(u_i) ={ u, v}, N(v_i) = {u, v} which implies that {N(u_i) ∩ N(v_i)}={u,v}=s=[n-(n-2)] and { N[u]∩ N[v]} $\neq V$. Hence S is an independent dominating set.

Theorem 2.24: Every graph G has an irredundant complete dominating set S then G≅ $H_1 + \cup_{i=1}^n H_i$ iff $|V-S| = n^2 + N(m+1) - 2$ and H_1 must have a irredundant complete dominating set

Proof: Given G be a graph with S is an irredundant complete dominating set If there exists vertices u and v such that uv$\notin E$ we have N[u]={u,$u_i, v_i, \cup_{i=1}^n w_i$} ,N[v]={v,$u_i, v_i, \cup_{i=1}^n w_i$}

case(i) suppose each $|w_i|$=1, we have N[u]-N[S-u]={u}$\neq \emptyset$ and N[u] ∩N[S-u]={ $u_i, v_i, \cup_{i=1}^n w_i$}=[V-S,W] where W= $\cup_{i=1}^n w_i$, then we have |N[u] ∩ N[S − u]| = $|V-S| + |W| = (n-2) + 2$ =2n-2=2(n-1) since H_1 contains n vertices and $\cup_{i=1}^n H_i$ contains n vertices .

Case(ii) Suppose each $|w_i|$=2 then we have |N[u] ∩ N[S − u]|=$|V-S| + |W| = (n-2) + 2n$ that is $|V-S| = n - 2 + 2n = 3n - 2$.Hence $|G| = |H_1 + \cup_{i=1}^n H_i|$, $|N_G[v]| = |N_G(S)| \cup |S|$ $|V-S| + |S|$=3n-2+2=3n.

Case(iii) Suppose each $|w_i|$ =2 then we have |N[u] ∩ N[S − u]| = $|V-S|+|W|=(n-2)+3n$ that is $|V-S|=n-2+3n=4n-2$.Hence $|G|=|H_1+\bigcup_{i=1}^{n} H_i|$, $|N_G[v]|=|N_G(S)|\cup|S|$, $|V-S|+|S|$=4n-2+2=4n.

Case(iv) Suppose each $|w_i|$ =m then we have |N[u] ∩ N[S − u]| = $|V-S|+|W|=(n-2)+mn$ that is $|V-S|=n-2+mn=n(m+1)-2$.Hence $|G|=|H_1+\bigcup_{i=1}^{n} H_i|$, $|N_G[v]|=|N_G(S)|\cup|S||V-S|+|S|$= $n(m+1)-2$+2= $n(m+1)$.

Case(v) Suppose each $|w_i|$=m+n then we have |N[u] ∩ N[S − u]|=$|V-S|+|W|=(n-2)+(m+n)n$ that is $|V-S|=n-2+(m+n)n=n^2+n(m+1)-2$.Hence $|G|=|H_1+\bigcup_{i=1}^{n} H_i|$,$|N_G[v]|=|N_G(S)|\cup|S|$ $|V-S|+|S|$= $n^2+n(m+1)-2$+2= $n^2+n(m+1)$.

Conversely, let us assume that number of vertices in V-S is $n^2+n(m+1)-2$ and we have to prove that G≅ $H_1+\bigcup_{i=1}^{n} H_i$, To prove G≅ $H_1+\bigcup_{i=1}^{n} H_i$, suppose a subgraph H_1 contains an irredundant complete dominating set S with $|V-S|$=H_1-S=n-S where $|H_1|$=n and H_1=$|V-S|$+S=n by adding every vertex of H_1 by an edge we have $|H_1|$=n+1, suppose adding every vertex of H_1 by two edges then we have $|H_1|$=n+2 such a way that we can add every vertex of H_1 by m number edges we have $|H_1|$=n+m and also by taking union of m number of vertices and n copies of such H_i ,that is subgraphs of H and taking union of n copies of H_i ,Hence we have G≅ $H_1+\bigcup_{i=1}^{n} H_i$.

Relationships with other Graph Theoretical Parameters:

Theorem 3.1: For any graph G=K_n –e with n≥3 vertices, $\gamma_{irn}(G)+\kappa(G)\le$ n and the bound is sharp iff G≅ k_n-e, for all n≥ 3.

Proof: Let G be a complete graph with K_n –e and n≥3 vertices. We know that $\kappa(G)\le(n-1)-1=(n-2)$ and by **theorem 2.21:** $\gamma_{irn}(G)\le[n-(n-2)]$. Hence n $\gamma_n(G)+\kappa(G)\le[n-(n-2)]+n-2=n$ .Suppose G is isomorphic to K_3 –e then clearly $\gamma_{irn}(G)+\kappa(G)=n$. Conversly, Let $\gamma_{irn}(G)+\kappa(G)=n$ this is possible only if $\gamma_{irn}(G)=[n-(n-2)]$ and $(G)=n-2$. Since $\kappa(G)=n-2$, G is isomorphic to K_n-e for which $\gamma_{irn}(G)=2=[n-(n-2)]$. Hence G≅ K_{n-e}, for all n≥ 3.

Theorem 3.2: For any graph G=K_n –e with n≥3 vertices, $\gamma_{irn}(G)+\chi(G)\le$ n+1 and the bound is sharp iff G≅ k_{n-e}, for all n≥ 3.

Proof: : Let G be a complete graph with Kn –e and n≥3 vertices. We know that $\chi(G)\le n-1$ and by **theorem2.21**$\gamma_{irn}(G)=[n-(n-2)$Hence $\gamma_{irn}(G)+\chi(G)\le[n-(n-2)]+n-1$that is $\gamma_{irn}(G)+\chi(G)=$ n + 1.Suppose G is isomorphic to K_3 –e then clearly$\gamma_{irn}(G)+\chi(G)=n+1$. Conversly, Let $\gamma_{irn}(G)+\chi(G)=n+1$ this is possible only if $\gamma_{irn}(G)=[n-(n-2)]$ and $\chi(G)=n-1$. Since $\chi(G)=n-1$, G is isomorphic to K_3 –e for which $\gamma_{irn}(G)=2=[n-(n-2)]$. Hence G≅ K_n-e, for all n≥ 3.

Theorem 3.3: For any graph G=Kn –e with n≥3 vertices, $\gamma_{irn}(G)+\Delta(G)\le n$ and the bound is sharp iff G≅ Kn – e , for all n≥3

Proof: Let G be n copies of complete graph with n≥3 vertices. We know that $\Delta(G)$ ≤n-1and by **theorem 2.21:** $\gamma_{irn}(G)\le[n-(n-2)]$, Hence $\gamma_{irn}(G)+\Delta(G)\le[n-(n-2)]$+n-1, that is $\gamma_{irn}(G)+\Delta(G)\le$ n+1 for k_n-e, for all n≥3, the bound is sharp.

Conclusion

In this paper we found an upper bound for the irredundant complete domination number and relationship between irredundant complete domination numbers of graphs and characterized the corresponding extremal graphs. Similarly irredundant complete domination numbers with other graph theoretical parameters can be considered.

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